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OPE of the energy-momentum tensor correlator in massless QCD

M. F. Zoller [a,1] and K. G. Chetyrkin [a,2],

[a] Institut für Theoretische Teilchenphysik, Karlsruhe Institute of Technology (KIT), D-76128 Karlsruhe, Germany

1: max.zoller@kit.edu
2: Konstantin.Chetyrkin@kit.edu

Abstract

We analytically calculate higher order corrections to coefficient functions of the operator product expansion (OPE) for the Euclidean correlator of two energy-momentum tensors in massless QCD. These are the three-loop contribution to the coefficient C_0 in front of the unity operator $O_0 = \mathbb{1}$ and the one and two-loop contributions to the coefficient C_1 in front of the gluon "condensate" operator $O_1 = -\frac{1}{4}G^{\mu\nu}G_{\mu\nu}$. For the correlator of two operators O_1 we present the coefficient C_1 at two-loop level (the coefficient function C_0 is known at four loops from [1]).

1 Motivation

The energy-momentum tensor correlator

$$T^{\mu\nu;\rho\sigma}(q) = i\int \mathrm{d}^4x\, e^{iqx}\, \langle 0|\hat{T}^{\mu\nu;\rho\sigma}(x)|0\rangle, \quad \hat{T}^{\mu\nu;\rho\sigma}(x) = T[T^{\mu\nu}(x)T^{\rho\sigma}(0)] \tag{1}$$

plays an important role in many physical problems. A lot of these lie in the field of Quark Gluon Plasma (QGP) physics. Here the correlator eq. (1) is the central object for describing transport properties, like the shear viscosity of the plasma (see e.g. [2, 3]) and spectral functions for some tensor channels in the QGP [4]. Another application is a sum rule approach to tensor glueballs. These special hadrons without valence quarks are determined by their gluonic degrees of freedom. QCD allows for such particles but a conclusive discovery has not yet been made.

In a sum rule approach [5] one usually starts with the vacuum correlator of an interpolating local operator which has the same quantum numbers as the hadrons we want to investigate. If we are interested in glueballs we take local operators consisting of gluon fields. For the cases $J^{PC} = 0^{++}, 0^{-+}$ and 2^{++} the following operators are usually considered:

$$O_1(x) = -\frac{1}{4}G^{\mu\nu}G_{\mu\nu}(x) \qquad \text{(scalar)} \tag{2}$$

$$\tilde{O}_1(x) = G^{\mu\nu}\tilde{G}_{\mu\nu}(x) \qquad \text{(pseudoscalar)} \tag{3}$$

$$O_T(x) = T^{\mu\nu}(x) \qquad \text{(tensor)} \tag{4}$$

where $G_{\mu\nu}$ is the gluon field strength tensor and

$$\tilde{G}_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma} G^{\rho\sigma} \tag{5}$$

the dual gluon field strength tensor. For more details see [6]. The vacuum expectation value (VEV) of the correlator of such a local operator $O(x)$

$$\Pi(Q^2) = i \int \mathrm{d}^4x\, e^{iqx}\, \langle 0|T[\,O(x)O(0)]|0\rangle \qquad (Q^2 = -q^2) \tag{6}$$

can of course be calculated in perturbation theory for large Euclidean momenta, but this is not enough. Starting from the perturbative region of momentum space we can probe into the non-perturbative region by means of an OPE. The idea originally formulated in [7] is to expand the non-local operator product $i \int \mathrm{d}^4x\, e^{iqx}\, T\,[O(x)O(0)]$ in a series of local operators with Wilson coefficients depending on the large Euclidean momentum q. In sum rules we usually have dispersion relations connecting the VEV of such a Euclidean operator product to some spectral density in the physical region of momentum space. As we are ultimately interested in the VEV of this operator product we only have to consider gauge invariant scalar operators in the expansion.

Effectively this expansion separates the high energy physics, which is contained in the Wilson coefficients, from the low energy physics which is taken into account by the VEVs of the local operators, the so-called condensates [5]. These cannot be calculated in perturbation theory, but need to be derived from low energy theorems or be calculated on the lattice. Such an OPE has already been done for the cases eq. (2) and eq. (3) (see [8, 9]) with one-loop accuracy.

In this work we present the results for the Wilson coefficients in front of the operators O_0 and $[O_1]$ for the correlator eq. (1) in massless QCD:

$$T^{\mu\nu;\rho\sigma}(q) \underset{q^2\to-\infty}{=\!=} C_0^{\mu\nu;\rho\sigma}(q)\mathbb{1} + C_1^{\mu\nu;\rho\sigma}(q)[O_1] + \ldots . \tag{7}$$

The brackets in $[O_1]$ indicate that we take a renormalized form of the operator O_1:

$$[O_1] = Z_G O_1^B = -\frac{Z_G}{4} G^{B\,\mu\nu} G^B_{\mu\nu} \tag{8}$$

where the index B marks bare quantities. We start our calculation with bare quantities which are expressed through renormalized ones in the end:

$$T^{\mu\nu;\rho\sigma}(q) = \sum_i C_i^{B\,\mu\nu;\rho\sigma}(q) O_i^B = \sum_i C_i^{\mu\nu;\rho\sigma}(q)[O_i]. \tag{9}$$

All physical matrix elements of $[O_1]$ are finite and so is the renormalized coefficient[1]

$$C_1 = \frac{1}{Z_G} C_1^B. \tag{10}$$

The renormalization constant

$$Z_G = 1 + \alpha_s \frac{\partial}{\partial \alpha_s} \ln Z_{\alpha_s} = \left(1 - \frac{\beta(\alpha_s)}{\varepsilon}\right)^{-1} \tag{11}$$

[1]This statement as well as eq. (9) are only true modulo so-called *contact terms*; see a detailed discussion in the next section.

has been derived in a simple way in [10] (see also an earlier work [11]). Here Z_{α_s} is the renormalization constant for α_s and we define[2]

$$\beta(\alpha_s) = \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \ln \alpha_s = -\sum_{i \geq 0} \beta_i \left(\frac{\alpha_s}{\pi}\right)^{i+1}. \tag{12}$$

In the massive case the two and four dimensional operators $O_f = m_f^2$ and $O_2^f = m_f \bar{\psi}_f \psi_f$ would have to be included as well for every massive quark flavour f. The VEVs of all other linearly independent scalar operators of dimension four vanish either by some equation of motion or as they are not gauge invariant. The contributions of higher dimensional operators are suppressed by higher powers of $\frac{1}{Q^2}$ in the coefficients.

Apart from the leading coefficient C_0 in front of the local operator $O_0 = \mathbb{1}$ the coefficient C_1 in front of $[O_1]$ is of special interest for many applications. One example is if we have a spectral density defined by our correlator and we want to calculate the shift in this spectral density from zero to finite temperature:

$$\Delta\rho(\omega, T) = \rho(\omega, T) - \rho(\omega, 0). \tag{13}$$

The spectral density at $T = 0$ is calculated from the VEV of the correlator whereas for the spectral density at finite T we take the thermal average of the operator product. For the unity operator $O_0 = \mathbb{1}$ the VEV and the thermal average are both 1 due to the normalization conditions. Hence the leading term from the OPE, i.e. the one proportional to O_0 vanishes in eq. (13) which makes the Wilson coefficients in front of O_1 and O_2^f the leading high frequency contributions to eq. (13). For more details see e.g. [12].

2 The energy-momentum tensor in QCD

The energy-momentum tensor which can be derived from the Lagrangian of a field theory is an interesting object by itself. To be identified with the physical object known from classical physics and general relativity it has to be symmetric as well as conserved. A very general method to derive such an energy-momentum tensor can be found e.g. in [13–15]. This has firstly been done for QCD in [16] and the result derived from the renormalized Lagrangian

$$\begin{aligned} \mathcal{L} = &-\frac{1}{4} Z_3\, G_{\mu\nu} G^{\mu\nu} - \frac{1}{2\lambda} \left(\partial_\mu A^\mu\right)^2 + \tilde{Z}_3 \partial_\rho \bar{c} \partial^\rho c + g_s \tilde{Z}_1 \partial_\rho \bar{c} \left(A^\rho \times c\right) \\ &+ \frac{i}{2} Z_2 \bar{\psi} \overleftrightarrow{\not{\partial}} \psi + g_s Z_{1\psi} \bar{\psi} \not{A} T \psi \end{aligned} \tag{14}$$

[2] Often in the literature Z_{α_s} is used instead of Z_G and $\alpha_s G^{\mu\nu} G_{\mu\nu}$ instead of O_1. This is justified because up to first order in α_s the renormalization constants Z_G and Z_{α_s} are the same. Only in higher orders Z_G and Z_{α_s} differ and therefore Z_G has to be used in such cases.

is

$$
\begin{aligned}
T_{\mu\nu} = & - Z_3 G_{\mu\rho} G_\nu^{\ \rho} + \frac{1}{\lambda}(\partial_\mu \partial_\rho A^\rho) A_\nu + \frac{1}{\lambda}(\partial_\nu \partial_\rho A^\rho) A_\mu \\
& + \tilde{Z}_3 (\partial_\mu \bar{c} \partial_\nu c + \partial_\nu \bar{c} \partial_\mu c) + g \tilde{Z}_1 \left(\partial_\mu \bar{c} (A_\nu \times c) + \partial_\nu \bar{c} (A_\mu \times c) \right) \\
& + \frac{i}{4} Z_2 \bar{\psi} \left(\overleftrightarrow{\partial_\mu} \gamma_\nu + \overleftrightarrow{\partial_\nu} \gamma_\mu \right) \psi + \frac{g}{2} Z_{1\psi} \bar{\psi} \left(A_\mu T \gamma_\nu + A_\nu \, T \gamma_\mu \right) \psi \\
& - g_{\mu\nu} \left\{ -\frac{1}{4} Z_3 \, G_{\rho\sigma} G^{\rho\sigma} + \frac{1}{\lambda} \left(\partial_\sigma \partial_\rho A^\rho \right) A^\sigma + \frac{1}{2\lambda} \left(\partial_\rho A^\rho \right)^2 \right. \\
& \left. + \tilde{Z}_3 \partial_\rho \bar{c} \partial^\rho c + g \tilde{Z}_1 \left(\partial_\rho \bar{c} (A_\rho \times c) \right) + \frac{i}{2} Z_2 \bar{\psi} \overleftrightarrow{\not{\partial}} \psi + g Z_{1\psi} \bar{\psi} \not{A} T \psi \right\},
\end{aligned}
\tag{15}
$$

Here

$$
G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \frac{\tilde{Z}_1}{\tilde{Z}_3} g_s \left(A_\mu \times A_\nu \right), \tag{16}
$$

$Z_3, \tilde{Z}_3$ and Z_2 stand for the field renormalization constants for the gluon, ghost and quark fields respectively and $\tilde{Z}_1$ and $Z_{1\psi}$ for the vertex renormalization constants. The abbreviation $(A_\mu \times A_\nu)^a = f^{abc} A^b_\mu A^c_\nu$, where f^{abc} is the structure constant of the SU(N_c) gauge group, is used and all colour indices are suppressed for convenience.
This energy-momentum tensor consists of gauge invariant as well as gauge and ghost terms. If we were to consider general matrix elements of operator products we would have to include all these terms. It has been pointed out in [16] however that for Green's functions with only gauge invariant operators it would be enough to take the gauge invariant part of the energy momentum tensor:

$$
\begin{aligned}
T_{\mu\nu}|_{\text{ginv}} = & - Z_3 G_{\mu\rho} G_\nu^{\ \rho} + \frac{i}{4} Z_2 \bar{\psi} \left(\overleftrightarrow{\partial_\mu} \gamma_\nu + \overleftrightarrow{\partial_\nu} \gamma_\mu \right) \psi + \frac{g}{2} Z_{1\psi} \bar{\psi} \left(A_\mu T \gamma_\nu + A_\nu \, T \gamma_\mu \right) \psi \\
& - g_{\mu\nu} \left\{ -\frac{1}{4} Z_3 \, G_{\rho\sigma} G^{\rho\sigma} + \frac{i}{2} Z_2 \bar{\psi} \overleftrightarrow{\not{\partial}} \psi + g Z_{1\psi} \bar{\psi} \not{A} T \psi \right\}.
\end{aligned}
\tag{17}
$$

This has been checked in our calculation of C_0 which we have done once with the full energy-momentum tensor eq. (15) and once with the gauge invariant part eq. (17) up to three-loop accuracy. As expected both calculations yield the same result.
The insertion of a local operator into a Green's function corresponds to an additional vertex in every possible Feynman diagram. For the energy-momentum tensor eq. (15) we get the vertices shown in Figure 1

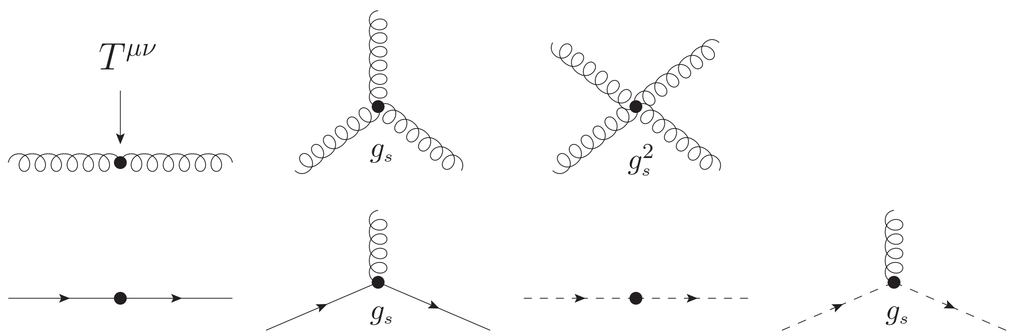


Figure 1: Energy-momentum tensor vertices and there dependence on g_s

In [16] it has been proven that the energy-momentum tensor of QCD is a finite operator which means the Z-factors appearing in (15) make any Green function of (renormalized) QCD elementary fields with *one* insertion of the operator $T_{\mu\nu}$ finite. We have used this theorem as a check for our setup and have calculated one - and two-loop corrections to the matrix elements $\langle \text{gluon(p},\mu_1)|T^{\mu\nu}|\text{gluon(p},\mu_2)\rangle$, $\langle \text{ghost(p)}|T^{\mu\nu}|\text{ghost(p)}\rangle$ and $\langle \text{quark(p), gluon(0},\mu_1)|T^{\mu}_{\mu}|\text{quark(p)}\rangle$ which turned out to be finite as expected.

Another important consequence of the finiteness property is the absence of the anomalous dimension of the energy-momentum tensor. For the bilocal operator $\hat{T}^{\mu\nu;\rho\sigma}(x)$ the situation is more complicated. This is because of extra (quartic!) UV divergences appearing in the limit of $x \to 0$. If x is kept away from 0 then $\hat{T}^{\mu\nu;\rho\sigma}(x)$ is finite and renormalization scheme independent. These divergences (which are local in x!) manifest themselves in the Fourier transform $T^{\mu\nu;\rho\sigma}(q)$. They can and should be renormalized with proper counterterms:

$$[T^{\mu\nu;\rho\sigma}(q)] = T^{\mu\nu;\rho\sigma}(q) - \sum_i Z_i^{ct}(q) O_i, \tag{18}$$

where O_i are some operators of (mass) dimension ≤ 4 and $Z_i^{ct}(q)$ are the corresponding (divergent) Z-factors. The latter must be *local*, that is have only *polynomial* dependence of the external momentum q. Within the $\overline{\text{MS}}$-scheme $Z_i^{ct}(q)$ are just poles in ε. It is of importance to note that the subtractive renormalization encoded in eq. (18) is in general *not* constrained by the QCD charge renormalization. Thus, the unambiguous QCD predictions for the coefficient functions in OPE (7) could be made only modulo contact terms proportional to $\delta(x)$ in position space.

3 OPE of the energy-momentum tensor correlator

The leading coefficient C_0 is just the perturbative VEV of the correlator eq. (1)

$$C_0^{\mu\nu;\rho\sigma}(q) = \langle 0|T^{\mu\nu;\rho\sigma}(q)|0\rangle|_{\text{pert}} \tag{19}$$

which we have computed up to order α_s^2 (three loops). In Figure 2 we show some sample Feynman diagrams contributing to this calculation. The energy-momentum tensor plays the role of an external current. In order to produce all possible Feynman diagrams we have used the program QGRAF [17]. As these diagrams are propagator-like the relevant integrals can be computed with the FORM package MINCER [18] after projecting them to scalar pieces. For the colour part of the diagrams the FORM package COLOR [19] has been used. Because of the four independent external Lorentz indices there are many possible tensor structures for the correlator eq. (1) and hence the Wilson coefficients. These are composed of the large external momentum q and the metric tensor g. Using the symmetries[3] of eq. (1) we can narrow them down to five possible independent tensor structures:

$$\begin{aligned}
t_1^{\mu\nu;\rho\sigma}(q) &= q^\mu q^\nu q^\rho q^\sigma, \\
t_2^{\mu\nu;\rho\sigma}(q) &= q^2 \ \left(q^\mu q^\nu g^{\rho\sigma} + q^\rho q^\sigma g^{\mu\nu}\right), \\
t_3^{\mu\nu;\rho\sigma}(q) &= q^2 \ \left(q^\mu q^\rho g^{\nu\sigma} + q^\mu q^\sigma g^{\nu\rho} + q^\nu q^\rho g^{\mu\sigma} + q^\nu q^\sigma g^{\mu\rho}\right), \\
t_4^{\mu\nu;\rho\sigma}(q) &= \left(q^2\right)^2 g^{\mu\nu} g^{\rho\sigma}, \\
t_5^{\mu\nu;\rho\sigma}(q) &= \left(q^2\right)^2 \left(g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}\right).
\end{aligned} \tag{20}$$

[3] These symmetries are $\mu \longleftrightarrow \nu$,$\rho \longleftrightarrow \sigma$ and $(\mu\nu) \longleftrightarrow (\rho\sigma)$.

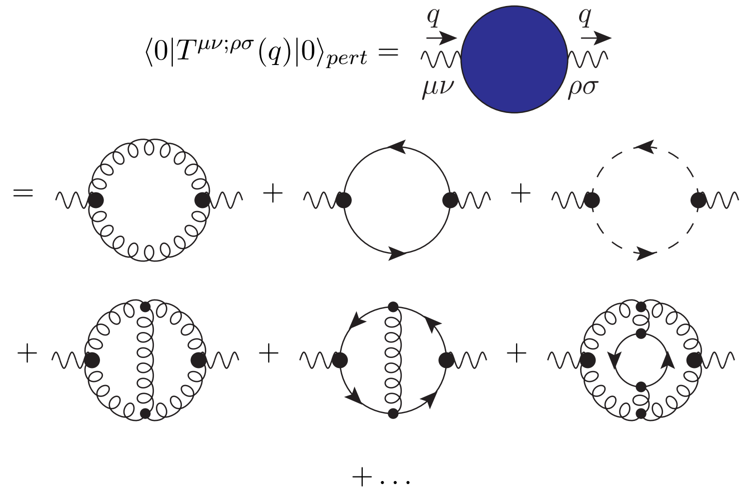


Figure 2: Diagrams for the calculation of the coefficient C_0

The conservation of the energy-momentum tensor leads to additional restrictions:

$$q_\mu T^{\mu\nu;\rho\sigma}(q) = \text{(local) contact terms} \tag{21}$$

This condition has been checked in all calculations. Subtracting the physically irrelevant contact terms leads to only two independent tensor structures which have also been used e.g. in [20]:

$$\begin{aligned} t_S^{\mu\nu;\rho\sigma}(q) =& \eta^{\mu\nu}\eta^{\rho\sigma} \\ t_T^{\mu\nu;\rho\sigma}(q) =& \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \frac{2}{D-1}\eta^{\mu\nu}\eta^{\rho\sigma} \\ \text{with} \quad \eta^{\mu\nu}(q) =& q^2 g^{\mu\nu} - q^\mu q^\nu \end{aligned} \tag{22}$$

where D is the dimension of the space time. The structure $t_T^{\mu\nu;\rho\sigma}(q)$ is traceless and orthogonal to $t_S^{\mu\nu;\rho\sigma}(q)$. Hence the latter corresponds to the part coming from the traces of the energy-momentum tensors. The Wilson coefficients in eq. (7) are then of the general form

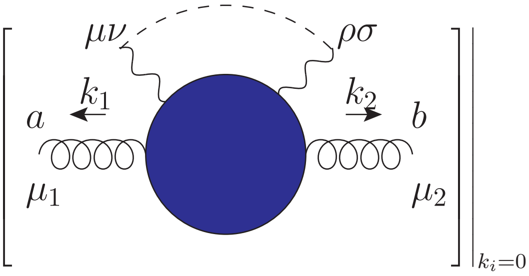
$$\begin{aligned} C_i^{\mu\nu;\rho\sigma}(q) &= \sum_{r=1,5} t_r^{\mu\nu;\rho\sigma}(q)\,(Q^2)^{\frac{-\dim(O_i)}{2}}\, C_i^{(r)}(Q^2) \\ &= \sum_{r=T,S} t_r^{\mu\nu;\rho\sigma}(q)\,(Q^2)^{\frac{-\dim(O_i)}{2}}\, C_i^{(r)}(Q^2) \;\; (+\text{ contact terms}). \end{aligned} \tag{23}$$

where $\dim(O_0) = 0$ and $\dim(O_1) = 4$ are the mass dimensions of the respective operators. The coefficients defined in the first line of eq. (23) and there conversion to the ones defined in the second line are given in the appendix. In order to compute the coefficient $C_1^{\mu\nu;\rho\sigma}(q)$ we have used the method of projectors [21] which allows to express coefficient functions for any OPE of two operators in terms of massless propagator type diagrams only. For the case of $C_1^{\mu\nu;\rho\sigma}(q)$ the following projector has been employed:

$$C_{1,B}^{\mu\nu;\rho\sigma}(q) = P_1\left(\iiint \mathrm{d}^4x\,\mathrm{d}^4y_1\,\mathrm{d}^4y_2\, e^{iqx+k_1y_1+k_2y_2}\,\langle 0|T[A_\mu^a(y_1)\,A_\nu^b(y_2)\,\hat{T}^{\mu\nu;\rho\sigma}(x)]|0\rangle^{\text{Q-irr}}\right). \tag{24}$$

Here

$$C^{\mu\nu;\rho\sigma}_{1,B}(q) = \frac{\delta^{ab}}{n_g}\frac{g^{\mu_1\mu_2}}{(D-1)}\frac{1}{D}\frac{\partial}{\partial k_1}\cdot\frac{\partial}{\partial k_2}\left[\; \cdots \;\right]\Bigg|_{k_i=0} \tag{25}$$



and the upper script $^{\mathrm{Q-irr}}$ means that only diagrams which become 1PI after formal gluing (depicted as a dotted line above) of two external lines carrying the (large) momentum q are included. Figure 3 shows some sample diagrams at tree and one-loop level.

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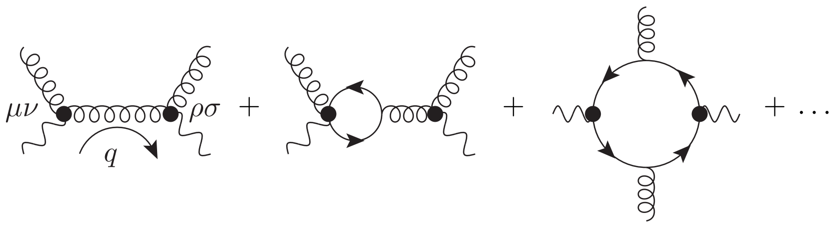


Figure 3: Diagrams for the calculation of the coefficient C_1.

4 Results

All results are given in the $\overline{\text{MS}}$ scheme with $a_s = \frac{\alpha_s}{\pi}$, $\alpha_s = \frac{g_s^2}{4\pi}$ and the abbreviation $l_{\mu q} = \ln\left(\frac{\mu^2}{Q^2}\right)$ where μ is the $\overline{\text{MS}}$ renormalization scale. They can be retrieved from
http://www-ttp.particle.uni-karlsruhe.de/Progdata/ttp12/ttp12-025/

The gauge group factors are defined in the usual way: C_F and C_A are the quadratic Casimir operators of the quark and the adjoint representation of the corresponding Lie algebra, d_R is the dimension of the quark representation, n_g is the number of gluons (dimension of the adjoint representation), T_F is defined so that $T_F\delta^{ab} = \mathbf{Tr}\left(T^aT^b\right)$ is the trace of two group generators of the quark representation.[4] For QCD (colour gauge group SU(3)) we have $C_F = 4/3$, $C_A = 3$, $T_F = 1/2$ and $d_R = 3$. By n_f we denote the number of active quark flavours.

4.1 C_0

Because of the contact terms both coefficients C_0^S and C_0^T could be unambiguously computed only up to constant (that is q-independent) contributions. To avoid the ambiguity we present

[4]For an SU(N) gauge group these are $d_R = N$, $C_A = 2T_FN$ and $C_F = T_F\left(N - \frac{1}{N}\right)$.

below their Q^2-derivatives:

$$
\begin{aligned}
Q^2 \frac{d}{dQ^2}\, C_0^{(T)} =& \frac{1}{16\pi^2}\Bigg[-\frac{1}{10} n_g - \frac{1}{20} n_f d_R \\
&+ a_s \left\{ \frac{1}{18} C_A n_g - \frac{7}{144} n_f T_F n_g \right\} \\
&+ a_s^2 \left\{ \frac{67}{12960} C_A^2 n_g + \frac{3}{128} n_f T_F C_F n_g - \frac{10663}{51840} n_f C_A T_F n_g + \frac{473}{6480} n_f^2 T_F^2 n_g \right. \\
&\quad + \frac{11}{216} l_{\mu q} C_A^2 n_g - \frac{109}{1728} l_{\mu q} n_f C_A T_F n_g + \frac{7}{432} l_{\mu q} n_f^2 T_F^2 n_g \\
&\quad \left. + \frac{11}{40} \zeta_3 C_A^2 n_g + \frac{3}{80} \zeta_3 n_f C_A T_F n_g - \frac{1}{20} \zeta_3 n_f^2 T_F^2 n_g \right\} \Bigg].
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
Q^2 \frac{d}{dQ^2}\, C_0^{(S)}(Q^2) =& \frac{a_s^2}{16\pi^2} \left\{ -\frac{121}{1296} C_A^2 n_g + \frac{11}{162} n_f C_A T_F n_g - \frac{1}{81} n_f^2 T_F^2 n_g \right\} \\
=& -\frac{a_s^2}{144\,\pi^2} \beta_0^2\, n_g,
\end{aligned} \tag{27}
$$

where

$$\beta_0 = \frac{11\, C_A}{12} - \frac{n_f\, T_f}{3}$$

is the first coefficient of the perturbative expansion of the β-function (12). This result for $Q^2 \frac{d}{dQ^2}\, C_0^{(T)}$ is in agreement with the one derived in [20] for the case of gluodynamics ($n_f = 0$) at order α_s (two-loop level). The simple form of eq. (27) comes from the well-known trace anomaly [16, 22], which reads

$$T_\mu^\mu = \frac{\beta(a_s)}{2}\, [G^a_{\rho\sigma} G^{a\,\rho\sigma}] = -2\,\beta(a_s)\,[O_1]. \tag{28}$$

Indeed, from *operator* eq. (28) we expect that

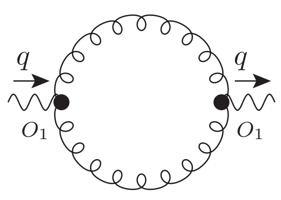
$$i \int \mathrm{d}^4 x\, e^{iqx}\, \langle 0 | T[\, T_\mu^\mu(x) T_\nu^\nu(0)] | 0 \rangle = 4\,\beta^2(\alpha_s)\, Q^4\, \Pi^{GG}(q^2) + \text{contact terms}, \tag{29}$$

where

$$Q^4\, \Pi^{GG}(q^2) = i \int \mathrm{d}^4 x\, e^{iqx}\, \langle 0 | T\,[O_1(x) O_1(0)] | 0 \rangle. \tag{30}$$

Now the one-loop result

$$Q^2 \frac{d}{dQ^2}\, \Pi^{GG}(q^2) =$$



$$= -\frac{1}{64\pi^2}\, n_g + \text{contact terms} \tag{31}$$

leads directly to eq. (27) The fact that this particular three-loop result can be derived from one-loop results is also the reason for the lack of ζ-functions in it. Furthermore the structure of eq. (28) explains nicely why the leading contribution for this scalar piece is of order α_s^2.

In fact, the correlator (30) is known in two-, three- and four-loop approximations from works [23],[24] and [1] respectively. The four-loop result reads (with all colour factors set to their QCD values and $l_{\mu q} = 0$)

$$
\begin{aligned}
Q^2 \frac{d}{dQ^2}\,\Pi^{GG}(q^2) = \frac{1}{16\pi^2}\Bigg\{ & - 2 + a_s\left(-\frac{73}{2} + \frac{7}{3} n_f\right) \\
& + a_s^2\left(-\frac{37631}{48} + \frac{495}{4}\zeta_3 + n_f\left[\frac{7189}{72} - \frac{5}{2}\zeta_3\right] - \frac{127}{54} n_f^2\right) \\
& + a_s^3\Bigg(-\frac{15420961}{864} + \frac{44539}{8}\zeta_3 - \frac{3465}{4}\zeta_5 \\
& + n_f\left[\frac{368203}{108} - \frac{11677}{24}\zeta_3 + \frac{95}{18}\zeta_5\right] \\
& + n_f^2\left[-\frac{115207}{648} + \frac{113}{12}\zeta_3\right] + n_f^3\left[\frac{7127}{2916} - \frac{2}{27}\zeta_3\right]\Bigg)\Bigg\}. \qquad (32)
\end{aligned}
$$

Finally, using eq. (29) and the well-known result for the four-loop QCD β-function [25, 26] we could easily extend the rhs of (27) by *three more orders* in α_s:

$$
\begin{aligned}
Q^2 \frac{d}{dQ^2}\,C_0^{(S)}(Q^2) = \frac{a_s^2}{16\,\pi^2}\Bigg\{ & -\frac{121}{18} + \frac{22}{27} n_f - \frac{2}{81} n_f^2 \\
+ \; a_s & \left(-\frac{11077}{72} + \frac{1025}{36} n_f - \frac{265}{162} n_f^2 + \frac{7}{243} n_f^2\right) \\
+ \; a_s^2 & \Bigg(-\frac{5787209}{1728} + \frac{6655}{16}\zeta_3 + n_f\left[\frac{540049}{648} - \frac{4235}{72}\zeta_3\right] + n_f^2\left[-\frac{556555}{7776} + \frac{275}{108}\zeta_3\right] \\
+ & \quad n_f^3\left[\frac{29071}{11664} - \frac{5}{162}\zeta_3\right] - \frac{127}{4374}\Bigg) \\
+ \; a_s^3 & \Bigg(-\frac{2351076745}{31104} + \frac{5925007}{288}\zeta_3 - \frac{46585}{16}\zeta_5 \\
+ \; n_f & \left[\frac{367411229}{15552} - \frac{33359777}{7776}\zeta_3 + \frac{240185}{648}\zeta_5\right] \\
+ & \quad n_f^2\left[-\frac{381988321}{139968} + \frac{3715127}{11664}\zeta_3 - \frac{12485}{972}\zeta_5\right] \\
+ \; n_f^3 & \left[\frac{20279497}{139968} - \frac{180083}{17496}\zeta_3 + \frac{95}{1458}\zeta_5\right] \\
& + n_f^4\left[-\frac{1101389}{314928} + \frac{427}{2916}\zeta_3\right] + n_f^5\left[\frac{7127}{236196} - \frac{2}{2187}\zeta_3\right]\Bigg)\Bigg\}. \qquad (33)
\end{aligned}
$$

4.2 C_1

According to the definition of $C_1^{\mu\nu;\rho\sigma}$ in eq. (23) there is a factor $\frac{1}{(Q^2)^2}$ in front of the dimensionless scalar pieces $C_1^{(S)}$ and $C_1^{(T)}$ which makes the whole coefficient immune to contact

terms except for those proportional to the tensor structures $t_4^{\mu\nu;\rho\sigma}(q)$ and $t_5^{\mu\nu;\rho\sigma}$ defined in eq. (20). The physical pieces $C_1^{(S)}$ and $C_1^{(T)}$ however are unambigous and the results read:

$$\begin{aligned} C_1^{(S)} = & \quad a_s \left\{ \frac{22}{27} C_A - \frac{8}{27} n_f T_F \right\} \\ & + \ a_s^2 \left\{ \frac{83}{324} C_A^2 - \frac{2}{9} n_f T_F C_F - \frac{8}{81} n_f C_A T_F - \frac{4}{81} n_f^2 T_F^2 \right\}, \end{aligned} \tag{34}$$

$$\begin{aligned} C_1^{(T)} = & \quad a_s \left\{ -\frac{5}{18} C_A - \frac{5}{72} n_f T_F \right\} \\ & + \ a_s^2 \left\{ -\frac{83}{432} C_A^2 + \frac{43}{96} n_f T_F C_F + \frac{41}{432} n_f C_A T_F - \frac{1}{216} n_f^2 T_F^2 \right\}. \end{aligned} \tag{35}$$

One thing to notice about $C_1^{(S)}$ is that if we take the trace of both energy-momentum tensors the whole term $\eta^{\mu}_{\ \mu}(q)\eta^{\nu}_{\ \nu}(q)\frac{1}{(Q^2)^2}C_1^{(S)}$ in the Wilson coefficient becomes local and, therefore, indistinguishable from contact terms. We can however check eq. (34) independently by computing first the coefficient function $C_1^{(TG,T)}$ in an OPE ($t_S^{\mu\nu} = q^4\, g_{\mu\nu}$, $t_T^{\mu\nu} = q^4\, g_{\mu\nu} - q^2 q_\mu\, q_\nu$)

$$\begin{aligned} & i \int \mathrm{d}^4 x\, e^{iqx}\, T[\, T^{\mu\nu}(x) G^2_{\rho\sigma}(0)] \underset{q^2 \to -\infty}{=\!=} \\ & \qquad \left(C_0^{(TG,S)} t_S^{\mu\nu} + C_0^{(TG,T)} t_T^{\mu\nu} \right) \mathbb{1} + \left(C_1^{(TG,S)} t_S^{\mu\nu} + C_1^{(TG,T)} t_T^{\mu\nu} \right) \frac{[O_1]}{Q^4} + \ldots \end{aligned} \tag{36}$$

and then employing eq. (28) to get the next higher order in α_s for $C_1^{(S)}$. The result

$$C_1^{(TG,T)} = -\frac{16}{3} + a_s \left(\frac{22}{9} C_A - \frac{8}{9} n_f T_F \right) + \mathcal{O}(\alpha_s^2) = -\frac{16}{3}\, (1 + \beta(a_s)/2) + \mathcal{O}(\alpha_s^2) \tag{37}$$

allows to represent the rhs of eq. (34) in a form directly confirming eq. (28):

$$C_1^S = \frac{\beta(\alpha_s)}{6}\, C_1^{(TG,T)} + \mathcal{O}(\alpha_s^3) = -\frac{8}{9}\, \beta(\alpha_s)\, (1 + \beta(a_s)/2) + \mathcal{O}(\alpha_s^3). \tag{38}$$

The factor $\beta(a_s)$ in this result is a direct consequence of the trace anomaly equation (28). However, we do not know any rationale behind the peculiar structure after this factor. If it is not accidental, then one can hope that an explanation could be found within the so-called β-expansion formalism suggested in [27].

It is important to note that the coefficient functions $C_1^{(S)}$ and $C_1^{(T)}$ are *not* Renormalization Group independent. We can construct the corresponding RG invariants by using the well-known fact[5] that the scale invariant version of the operator O_1 is

$$O_1^{RGI} \equiv \hat{\beta}(a_s)\, [O_1], \quad \hat{\beta}(a_s) = \frac{-\beta(a_s)}{\beta_0} = a_s \left(1 + \sum_{i \geq 1} \frac{\beta_i}{\beta_0} a_s^i \right). \tag{39}$$

From this and the scale invariance of $T^{\mu\nu;\rho\sigma}(q)$ defined in eq. (1) we find the RG invariant Wilson coefficients

$$\begin{aligned} C_{1,RGI}^{(S)} &\equiv C_1^{(S)} / \hat{\beta}(a_s) \\ C_{1,RGI}^{(T)} &\equiv C_1^{(T)} / \hat{\beta}(a_s) \end{aligned} \tag{40}$$

[5] This follows directly from the RG invariance of the energy-momentum tensor and the trace anomaly equation (28).

which satisfy

$$C_{1,RGI}^{(S,T)} O_1^{RGI} = C_1^{(S,T)}[O_1]. \tag{41}$$

From this definition we can immediately explain the absence of $l_{\mu q}$ in eq. (34) and eq. (35). Suppose we had $l_{\mu q}$ in $C_1^{(S)}$ and therefore in $C_{1,RGI}^{(S)}$ then the general structure of eq. (40) up to three-loop order would be

$$\begin{aligned} C_{1,RGI}^{(S,T)} &= (a_1 + b_1\, l_{\mu q}) + a_s(a_2 + b_2\, l_{\mu q} + c_2\, l_{\mu q}^2) \\ &\quad + a_s^2(a_3 + b_3\, l_{\mu q} + c_3\, l_{\mu q}^2 + d_3\, l_{\mu q}^3) + \mathcal{O}(a_s^3) \end{aligned} \tag{42}$$

with scale independent coefficients a_i, b_i, c_i and d_i. The derivative with respect to μ^2 must vanish:

$$\begin{aligned} \mu^2 \frac{d}{d\mu^2} C_{1,RGI}^{(S,T)} &= b_1 + a_s(b_2 + 2c_2\, l_{\mu q}) + a_s\beta(a_s)(a_2 + b_2\, l_{\mu q} + c_2\, l_{\mu q}^2) \\ &\quad + a_s^2(b_3 + 2c_3\, l_{\mu q} + 3d_3\, l_{\mu q}^2) + \mathcal{O}(a_s^3) \stackrel{!}{=} 0 \quad \forall\, \mu^2 \\ &\Rightarrow b_1 = 0 \\ &\Rightarrow b_2 = 0,\; c_2 = 0 \\ &\Rightarrow b_3 = \beta_0 a_2,\; c_3 = 0,\; d_3 = 0. \end{aligned} \tag{43}$$

In conclusion, not only have we explained the absence of logarithms in eq. (34) and eq. (35) but we also get the logarithmic part of the three-loop result for these coefficient functions for free. Terms with $l_{\mu q}^2$ can only appear starting from four-loop level, terms with $l_{\mu q}^3$ from five-loop level and so on.
The quantities defined in eq. (40) are given by

$$C_{1,RGI}^{(S)} = \tfrac{22}{27} C_A - \tfrac{8}{27} n_f T_F - \tfrac{a_s}{324}\left(11C_A - 4n_f T_F\right)^2 \tag{44}$$

$$\begin{aligned} C_{1,RGI}^{(T)} = & -\tfrac{5}{72}(4C_A + n_f T_F) + \tfrac{a_s}{864(11C_A - 4n_f T_F)}\left(214C_A^3 + 876C_A^2 n_f T_F\right. \\ & \left. + 3537 C_A C_F n_f T_F - 672 C_A n_f^2 T_F^2 - 1728 C_F n_f^2 T_F^2 + 16 n_f^3 T_F^3\right) \end{aligned} \tag{45}$$

The three-loop parts proportional to $l_{\mu q}$ are

$$C_{1,RGI}^{(S,\mathrm{3l,log})} = a_s^2 l_{\mu q} \left\{ -\tfrac{1331 C_A^3}{3888} + \tfrac{121}{324} C_A^2 n_f T_F - \tfrac{11}{81} C_A n_f^2 T_F^2 + \tfrac{4}{243} n_f^3 T_F^3 \right\}, \tag{46}$$

$$C_{1,RGI}^{(T,\mathrm{3l,log})} = a_s^2 l_{\mu q} \left\{ \tfrac{214C_A^3 + 876C_A^2 n_f T_F + 3537 C_A C_F n_f T_F - 672 C_A n_f^2 T_F^2 - 1728 C_F n_f^2 T_F^2 + 16 n_f^3 T_F^3}{10368} \right\}. \tag{47}$$

For completeness we have also computed the contribution of the gluon condensate to the OPE of correlator (30):

$$Q^4\, \Pi^{GG}(q^2) \underset{q^2 \to -\infty}{=\!=\!=} C_0^{GG}\, Q^4 \quad + \quad C_1^{GG}\, \langle 0|[O_1]|0\rangle \tag{48}$$

with the result:

$$
\begin{aligned}
C_1^{GG} =& -1 + a_s\left(-\frac{49}{36}C_A + \frac{5}{9}n_f T_F - \frac{11}{12}l_{\mu q}C_A + \frac{1}{3}l_{\mu q}n_f T_F\right)\\
&+ a_s^2\left(-\frac{11509}{1296}C_A^2 + \frac{13}{4}n_f T_F C_F + \frac{3095}{648}n_f C_A T_F - \frac{25}{81}n_f^2 T_F^2\right.\\
&-\frac{1151}{216}l_{\mu q}C_A^2 + l_{\mu q}n_f T_F C_F + \frac{97}{27}l_{\mu q}n_f C_A T_F - \frac{10}{27}l_{\mu q}n_f^2 T_F^2\\
&-\frac{121}{144}l_{\mu q}^2 C_A^2 + \frac{11}{18}l_{\mu q}^2 n_f C_A T_F - \frac{1}{9}l_{\mu q}^2 n_f^2 T_F^2 + \frac{33}{8}\zeta_3 C_A^2\\
&\left.-3\zeta_3 n_f T_F C_F + \frac{3}{2}\zeta_3 n_f C_A T_F\right)\\
&+ \boxed{\frac{a_s^2}{\varepsilon}\left(-\frac{17}{24}C_A^2 + \frac{1}{4}n_f T_F C_F + \frac{5}{12}n_f C_A T_F\right)}\,.
\end{aligned}
\tag{49}
$$

The tree and one-loop contributions in (49) are in agreement with [8] and [28, 29] correspondingly. The two-loop part is new and has a feature that did not occur in lower orders, namely, a divergent contact term. Its appearance clearly demonstrates that non-logarithmic perturbative contributions to C_1^{GG} are *not* well defined in QCD, a fact seemingly ignored by the QCD sum rules practitioners (see, e.g. [6, 30]).

An unambiguous QCD prediction can be made for the derivative:

$$
\begin{aligned}
Q^2\frac{d}{dQ^2}\,C_1^{GG} =& a_s\left(\frac{11}{12}C_A - \frac{1}{3}n_f T_F\right)\\
&+ a_s^2\left(\frac{1151}{216}C_A^2 - n_f T_F C_F - \frac{97}{27}n_f C_A T_F + \frac{10}{27}n_f^2 T_F^2\right.\\
&\left.+\frac{121}{72}l_{\mu q}C_A^2 - \frac{11}{9}l_{\mu q}n_f C_A T_F + \frac{2}{9}l_{\mu q}n_f^2 T_F^2\right)\,.
\end{aligned}
\tag{50}
$$

5 Numerics

In this section we will give our main results in the numerical form for two cases of interest, that is gluodynamics ($n_f = 0$) and QCD with three light quarks only ($n_f = 3$). As has already been mentioned, not all coefficient functions which we have discussed in the previous section are Renormalization Group independent. For a meaningful discussion we will construct the corresponding RG invariants by using the scale invariant version of the operator O_1 defined in eq. (39). In addition we set $l_{\mu q} = 0$ everywhere.[6]

$$
Q^2\frac{d}{dQ^2}\,C_0^{(T)} \underset{n_f=0}{=\!=} -\frac{4}{80\pi^2}\left(1 - 1.66667\,a_s - 30.2162\,a_s^2\right), \tag{51}
$$

$$
Q^2\frac{d}{dQ^2}\,C_0^{(T)} \underset{n_f=3}{=\!=} -\frac{5}{64\pi^2}\left(1 - 0.6\,a_s - 15.1983\,a_s^2\right), \tag{52}
$$

$$
Q^2\frac{d}{dQ^2}\,C_0^{(S)} \underset{n_f=0}{=\!=} -\frac{121}{288\pi^2}a_s^2\left(1 + 22.8864a_s + 423.833a_s^2 + 8014.74a_s^3\right), \tag{53}
$$

$$
Q^2\frac{d}{dQ^2}\,C_0^{(S)} \underset{n_f=3}{=\!=} -\frac{9}{32\pi^2}a_s^2\left(1 + 18.3056a_s + 247.48a_s^2 + 3386.41a_s^3\right), \tag{54}
$$

[6]This corresponds to the choice $\mu^2 = Q^2$ for the renormalization scale.

$$Q^2 \frac{d}{dQ^2} C_1^{GG,RGI} \underset{n_f=0}{=\!=} \frac{11}{4} a_s^2 \, (1 + 19.7576 \, a_s) \,, \quad C_1^{GG,RGI} \equiv \hat{\beta}(a_s) \, C_1^{GG}, \tag{55}$$

$$Q^2 \frac{d}{dQ^2} C_1^{GG,RGI} \underset{n_f=3}{=\!=} \frac{9}{4} a_s^2 \, (1 + 15.3889 \, a_s) \,, \tag{56}$$

$$C_{1,RGI}^{(S)} \underset{n_f=0}{=\!=} \frac{22}{9} \, (1 - 1.375 \, a_s) \,, \quad C_{1,RGI}^{(S)} \equiv C_1^{(S)} / \hat{\beta}(a_s), \tag{57}$$

$$C_{1,RGI}^{(S)} \underset{n_f=3}{=\!=} 2 \, (1 - 1.125 \, a_s) \,, \tag{58}$$

$$C_{1,RGI}^{(T)} \underset{n_f=0}{=\!=} -\frac{5}{6} \, (1 - 0.2431825 \, a_s) \,, \quad C_{1,RGI}^{(T)} \equiv C_1^{(T)} / \hat{\beta}(a_s), \tag{59}$$

$$C_{1,RGI}^{(T)} \underset{n_f=3}{=\!=} -\frac{15}{16} \, (1 - 1.3333 \, a_s) \,. \tag{60}$$

6 Applications to high-temperature QCD

Recently, the correlators Π^{GG} and $T^{\mu\nu;\rho\sigma}(q)$ have been studied in (Euclidean) hot Yang-Mills theory in [31, 32] respectively (see, also references therein for related earlier works).

In this section we will employ our $\mathbf{T} = 0$ calculations in order to extend *some* of the results of these publications by adding fermionic contributions as well as higher order corrections. Note that for simplicity we will set all colour factors in all expressions below to their QCD values. The reader interested in expressions valid for generic colour group should be able to derive the corresponding results himself from our results.

6.1 Trace anomaly correlator

In [31] two-loop corrections to the quantity[7]

$$G_\theta(X) \equiv \langle T[\theta(X) \, \theta(0)] \rangle_c, \quad \theta \equiv T^\mu_\mu, \tag{61}$$

where $\langle \dots \rangle_c$ stands for the connected part and the expectation value is taken at finite temperature[8] $\mathbf{T}$, have been computed. The capital case X for the space-time argument in (61) is used in order to stress that we are dealing with a Euclidean correlator. In the following e and $p = |\mathbf{q}|$ are the energy and momentum densities with the well-known relation $\langle \theta \rangle_c = e - 3p$. In the limit of small $r \equiv |X|$ the result of [31] reads [9]

$$\frac{4 \, a_s^2}{\beta^2(a_s)} G_\theta(r) = \frac{384}{\pi^4 r^8} \, \bar{\gamma}_{\theta;\,\mathbb{1}}(r) \; - \; \frac{8 \, a_s \, \langle \theta \rangle_c}{\beta(a_s) \pi^2 r^4} \, \bar{\gamma}_{\theta;\,\theta}(r) \; - \; \frac{64(e+p)}{\pi^2 r^4} \, \bar{\gamma}_{\theta;\,e+p}(r) \; + \; \mathcal{O}\left(\frac{\mathbf{T}^6}{r^2}\right), \tag{62}$$

[7]Note that $G_\theta(0, \vec{X})$ has been directly measured in lattice simulations [33].

[8]We use the bold case for the temperature to make it distinct from $T(\dots)$ standing for the time ordered product of operators inside the round brackets.

[9]The expression below is the somewhat modified eq. (5.7) of [31].

with

$$\bar{\gamma}_{\theta;\mathbb{1}}(r) = a_s^2 + a_s^3\left(-\frac{1}{12} + \frac{11}{2}\,l_{\mu X}\right) + \mathcal{O}(a_s^4), \tag{63}$$

$$\bar{\gamma}_{\theta;\theta}(r) = 22\,a_s^2 + \mathcal{O}(a_s^3), \tag{64}$$

$$\bar{\gamma}_{\theta;e+p}(r) = a_s^2 + a_s^3\left(\frac{15}{72} + \frac{11}{2}\,l_{\mu X}\right) + \mathcal{O}(a_s^4), \tag{65}$$

and $l_{\mu X} = \log(\mu^2 X^2/4) + 2\gamma_E$.

According to [34] the coefficient functions $\bar{\gamma}_{\theta;\mathbb{1}}$ and $\bar{\gamma}_{\theta;\theta}(r)$ do not depend on temperature $\mathbf{T}$ and, thus, should coincide with their $\mathbf{T} = 0$ counterparts. Hence, we can use our momentum space results described in previous sections to arrive at the following QCD predictions for both coefficient functions[10].

$$\begin{aligned}
\bar{\gamma}_{\theta;\mathbb{1}}(r) = {}& a_s^2 + a_s^3\left(-\frac{1}{12} + \frac{11}{2}\,l_{\mu X} + n_f\left[-\frac{1}{18} - \frac{1}{3}\,l_{\mu X}\right]\right) + a_s^4\left(-\frac{49}{24} - \frac{495}{8}\zeta_3 + \frac{397}{16}\,l_{\mu X}\right. \\
+ {}& \left.\frac{363}{16}\,l_{\mu X}^2 + n_f\left[-\frac{35}{144} + \frac{5}{4}\zeta_3 - \frac{43}{12}\,l_{\mu X} - \frac{11}{4}\,l_{\mu X}^2\right] + n_f^2\left[-\frac{13}{216} + \frac{1}{36}\,l_{\mu X} + \frac{1}{12}\,l_{\mu X}^2\right]\right) \\
+ {}& a_s^5\left(-\frac{255155}{1728} - \frac{2915}{16}\zeta_3 + \frac{3465}{8}\zeta_5 + \frac{20891}{192}\,l_{\mu X} - \frac{5445}{8}\zeta_3\,l_{\mu X} + \frac{1793}{8}\,l_{\mu X}^2 + \frac{1331}{16}\,l_{\mu X}^3\right. \\
+ {}& n_f\left[\frac{38741}{1728} - \frac{9}{16}\zeta_3 - \frac{95}{36}\zeta_5 - \frac{16685}{576}\,l_{\mu X} + 55\zeta_3\,l_{\mu X} - \frac{4241}{96}\,l_{\mu X}^2 - \frac{121}{8}\,l_{\mu X}^3\right] \\
+ {}& n_f^2\left[-\frac{361}{216} + \frac{125}{72}\zeta_3 + \frac{491}{1728}\,l_{\mu X} - \frac{5}{6}\zeta_3\,l_{\mu X} + \frac{289}{144}\,l_{\mu X}^2 + \frac{11}{12}\,l_{\mu X}^3\right] \\
+ {}& \left.n_f^3\left[\frac{37}{1458} - \frac{1}{27}\zeta_3 + \frac{13}{324}\,l_{\mu X} - \frac{1}{108}\,l_{\mu X}^2 - \frac{1}{54}\,l_{\mu X}^3\right]\right) + \mathcal{O}(a_s^6),
\end{aligned} \tag{66}$$

$$\begin{aligned}
\bar{\gamma}_{\theta;\theta}(r) = {}& a_s^2\left(22 - \frac{4}{3}\,n_f\right) \\
+ {}& a_s^3\left(+\frac{788}{3} + 121\,l_{\mu X} + n_f\left[-\frac{304}{9} - \frac{44}{3}\,l_{\mu X}\right] + n_f^2\left[\frac{8}{27} + \frac{4}{9}\,l_{\mu X}\right]\right) + \mathcal{O}(a_s^4).
\end{aligned} \tag{67}$$

Note that our *vacuum* calculations produce no information about the coefficient function $\bar{\gamma}_{\theta;e+p}$ corresponding to the *traceless* part of the energy-momentum tensor.

Numerically eqs. (66) and (67) read (we set $l_{\mu X} = 0$)

$$\bar{\gamma}_{\theta;\mathbb{1}}(r) \underset{n_f=0}{=\!=} a_s^2 - 0.08333\,a_s^3 - 76.4189\,a_s^4 + 82.4604a_s^5 + \mathcal{O}(a_s^6), \tag{68}$$

$$\bar{\gamma}_{\theta;\mathbb{1}}(r) \underset{n_f=3}{=\!=} a_s^2 - 0.25\,a_s^3 - 73.1821\,a_s^4 + 142.705a_s^5 + \mathcal{O}(a_s^6), \tag{69}$$

$$\bar{\gamma}_{\theta;\theta}(r) \underset{n_f=0}{=\!=} 22\left(a_s^2 + 11.9394\,a_s^3\right) + \mathcal{O}(a_s^4), \tag{70}$$

[10] The details of the corresponding Fourier transformation are spelled e.g. in [35].

$$\bar{\gamma}_{\theta;\theta}(r) \underset{n_f=3}{=\!=\!=} 18\left(a_s^2 + 9.11111\,a_s^3\right) + \mathcal{O}(a_s^4). \tag{71}$$

6.2 Shear stress correlator

In [32] the so-called shear stress correlator, defined as

$$G_\eta(X) = -16\,c_\eta^2\,\langle T[T^{12}(X)\,T^{12}(0)]\rangle_c \tag{72}$$

with $X = (X_0, \vec{X}), \vec{X} = (0, 0, X_3)$, has been computed up to two-loops in high-temperature Yang-Mills theory. Here c_η is an arbitrary constant (introduced for some reason that is not quite clear to us in [32]) which we put for simplicity equal to i/4. The calculation has been performed with the help of an ultraviolet expansion valid in the limit of small distances or large momenta; the result has been presented in the form of an OPE. As the corresponding Wilson coefficients should be **T**-independent the results of [32] can be checked and extended further with the help of our calculations.[11]

We start from momentum space. In the zero temperature limit the function

$$\tilde{G}_\eta(Q^2) = \int \mathrm{d}^4 X\, e^{iQX}\, G_\eta(X)$$

is related to contribution to energy-momentum tensor correlator (1) proportional to the tensor structure $t_5^{\mu\nu;\rho\sigma}(q)$. This fact could be easily checked by applying projector (2.5) of [32] to the correlator $T^{\mu\nu;\rho\sigma}(q)$ expressed in terms of five independent tensor structures displayed in (20). The result reads $-8\,Q^4\,(1 - 7/2\,\varepsilon + 7/2\,\varepsilon^2 - \varepsilon^3)\,\left(C_{\mathbb{1}}^5(Q^2) + C_\theta^5(Q^2)\,\frac{\langle 0|\theta|0\rangle}{Q^4} + \dots\right)$.

Thus, we will work with the representation

$$T^{\mu\nu;\rho\sigma}(q) \underset{q^2\to-\infty}{=\!=\!=} t_5^{\mu\nu;\rho\sigma}(q)\left(C_{\mathbb{1}}^5(Q^2) + C_\theta^5(Q^2)\,\frac{\langle 0|\theta|0\rangle}{Q^4} + \dots\right) + \text{ structures 1-4} \tag{73}$$

We first concentrate on the coefficient function $C_\theta^5(Q^2)$ as the two-loop expression for $C_{\mathbb{1}}^5(Q^2)$ presented in [32] is in agreement to the previously known expression obtained in [20]. The result of [32] for the second term in eq. (73) reads:

$$C_\theta^5(Q^2) \quad = \quad -\frac{1}{3\beta_0\,a_s}\left(1 - \frac{\beta_0\,a_s}{4}\ln\zeta_{12}\right), \tag{74}$$

where ζ_{12} is an unknown constant. Note that the second term of the above expression is obtained not from a calculation but with the use of Renormalization Group considerations similar to those leading to eq. (43). Such a derivation assumes that the coefficient function C_θ^5 is finite which is not obvious as the corresponding Feynman integrals have logarithmic

[11] The Wilson coefficients in front of Lorentz non-invariant operators are for the moment not reachable with our projectors. It would be interesting however to extend these methods in order to reach e.g. the coefficient in front of $\langle T^{00}\rangle \sim e + p$ with a similar approach.

divergences stemming from the region of small x in eq. (1). Our direct calculation explicitly demonstrates the presence of such divergences:

$$
\begin{aligned}
C_\theta^{(5)}(Q^2) &= \frac{1}{3\,\beta(a_s)}\left\{1 + a_s\left(\frac{41}{24} + \frac{7}{288}\,n_f\right) + a_s^2\left(\frac{117}{32} - \frac{457}{576}\,n_f + \frac{1}{576}\,n_f^2\right)\right.\\
&+ \left.\frac{a_s}{\varepsilon}\left(\frac{11}{4} - \frac{1}{6}\,n_f\right) + \frac{a_s^2}{\varepsilon}\left(\frac{51}{8} - \frac{19}{24}\,n_f\right)\right\}.
\end{aligned}
\tag{75}
$$

It is important to note that the contribution proportional to C_θ^5 in (73) contains contact terms *only*. This is in agreement with (35) due to an identity

$$
C_1^{(T)} - C_1^{(5)} = \text{contact terms}, \tag{76}
$$

which, in turn, follows from restriction (21) (recall that $C_\theta^{(5)}(Q^2) \equiv C_1^{(5)}/(-2\,\beta(a_s))$ as a consequence of (28)).

In Euclidean position space eq. (73) can be presented as follows:

$$
\hat{T}^{\mu\nu;\rho\sigma}(X) \underset{r\to 0}{=\!=} \left(\delta^{\mu\rho}\delta^{\nu\sigma} + \delta^{\mu\sigma}\delta^{\nu\rho}\right)\left\{\tilde{C}_{\mathbb{1}}^5(r)\,\mathbb{1} + \tilde{C}_\theta^5(r)\,\langle 0|\theta|0\rangle + \dots\right\} + \text{ structures 1-4} \tag{77}
$$

Eq. (76), rewritten in terms of RG invariant quantities assumes the form:

$$
C_{1,RGI}^{(T)} - 2\,\beta_0\,C_\theta^{(5)} = \text{contact terms}. \tag{78}
$$

By recalling that the contact terms do not contribute the function $G_\eta(x)$ for all $x \neq 0$ we conclude that eqs. (75) and (47) contain all information to construct the first non-zero term $\mathcal{O}(a_s^2)$ in the coefficient function $\tilde{C}_\theta^5(x)$ with the result

$$
2\,\beta_0\tilde{C}_\theta^5(r) = \frac{a_s^2}{\pi^2\,r^4}\left(\frac{107}{192} + \frac{17}{16}\,n_f - \frac{5}{48}\,n_f^2 + \frac{1}{5184}\,n_f^3\right) \tag{79}
$$

Finally, using the identity

$$
C_0^{(T)} - C_0^{(5)} = \text{contact terms}, \tag{80}
$$

and (26) we arrive at the following result

$$
\begin{aligned}
\tilde{C}_{\mathbb{1}}^5(x) &= \frac{1}{\pi^4\,r^8}\left\{\frac{48}{5} + \frac{9}{5}\,n_f + a_s\left(-16+\frac{7}{3}\,n_f\right) + a_s^2\left(\frac{711}{5} - \frac{1188}{5}\zeta_3 - 44\,l_{\mu X}\right.\right.\\
&+ \left.\left. n_f\left[-\frac{259}{120} - \frac{27}{5}\zeta_3 + \frac{109}{12}\,l_{\mu X}\right] + n_f^2\left[-\frac{41}{90} + \frac{6}{5}\zeta_3 - \frac{7}{18}\,l_{\mu X}\right]\right)\right\}.
\end{aligned}
\tag{81}
$$

Numerical versions of eqs. (79) and (81) with $l_{\mu X} = 0$ are presented below.

$$
2\,\beta_0\,\tilde{C}_\theta^5(x) \underset{n_f=0}{=\!=} \frac{a_s^2}{\pi^2\,r^4}\left\{\frac{107}{192} = 0.557292\right\}, \tag{82}
$$

$$
2\,\beta_0\,\tilde{C}_\theta^5(x) \underset{n_f=3}{=\!=} \frac{a_s^2}{\pi^2\,r^4}\left\{\frac{45}{16} = 2.81250\right\}, \tag{83}
$$

$$\tilde{C}^5_{\mathbb{1}}(x) \underset{n_f=0}{=\!=} \frac{48}{5}\frac{1}{\pi^4 r^8}\left(1 - 1.66667\, a_s - 14.9384\, a_s^2\right), \tag{84}$$

$$\tilde{C}^5_{\mathbb{1}}(x) \underset{n_f=3}{=\!=} \frac{15}{\pi^4 r^8}\left(1 - 0.6\, a_s - 10.6983\, a_s^2\right). \tag{85}$$

7 Discussion and Conclusions

We have presented higher order corrections to coefficient functions C_0 and C_1 of the OPE of two energy-momentum tensors in massless QCD as well as for the OPE of two scalar "gluon condensate" operators in massless QCD. Our results extend the previously known accuracy by one loop for the coefficient functions in front of the unit operator and by two loops for the CF of the gluon condensate operator $O_1 = -\frac{1}{4}G^{\mu\nu}G_{\mu\nu}$.

We have confirmed all previously available results and in some cases extended them from purely Yang-Mills theory to QCD. Contrary to previous assumptions, we have found that the coefficient functions C_1^{GG} as well as $C_\theta^{(5)}(Q^2)$ are not completely finite with the standard QCD renormalization.

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In conclusion we want to mention that all our calculations have been performed on a SGI ALTIX 24-node IB-interconnected cluster of 8-cores Xeon computers using the thread-based [36] version of FORM [37]. The Feynman diagrams have been drawn with the Latex package Axodraw [38].

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A Results for $C_0^{(r)}$ and $C_1^{(r)}$, $r = 1\ldots 5$ and conversion to $C_0^{(S,T)}$ and $C_1^{(S,T)}$

Here we give our intermediate results for the coefficients $C_0^{(r)}$ and $C_1^{(r)}$ $(r = 1\ldots 5)$ appearing in the first line of eq. (23), i.e. the coefficients for the tensor structures eq. (20) before subtraction of contact terms:

$$C_0^{(r)} = C_0^{B\,(r)}, \tag{86}$$

$$C_1^{(r)} = \frac{1}{Z_G^2} C_1^{B\,(r)}. \tag{87}$$

The conservation of the energy-momentum tensor in classical field theory translates to

$$\partial_\mu T^{\mu\nu} = \text{local terms [16]}. \tag{88}$$

From this we get the relation

$$q_\mu C_i^{\mu\nu;\rho\sigma}(q) = \text{(local) contact terms} \quad (i = 0, 1) \tag{89}$$

which leads to the three restrictions serving as checks in our calculation:

$$\begin{aligned}
D_i^{(1)}(Q^2) &:= C_{i,1}(Q^2) + C_{i,2}(Q^2) + 2\,C_{i,3}(Q^2) = \text{(local) contact terms},\\
D_i^{(2)}(Q^2) &:= C_{i,2}(Q^2) + C_{i,4}(Q^2) = \text{(local) contact terms},\\
D_i^{(3)}(Q^2) &:= C_{i,3}(Q^2) + C_{i,5}(Q^2) = \text{(local) contact terms}.
\end{aligned} \tag{90}$$

Hence the subtraction of contact terms enables us to write the Wilson coefficients in terms of only two independent tensor structures eq. (22) which are related to the original five by the following equations:

$$\begin{aligned}
C_i^{(S)}(Q^2) &= -C_{i,2}(Q^2) - \frac{2}{(D-1)} C_{i,3}(Q^2),\\
C_i^{(T)}(Q^2) &= -C_{i,3}(Q^2).
\end{aligned} \tag{91}$$

A.1 $C_0^{(r)}$, $r = 1 \ldots 5$

These coefficients fulfill the relations eq. (90) even without local terms:

$$\begin{aligned}
D_0^{(1)}(Q^2) &:= C_{0,1}(Q^2) + C_{0,2}(Q^2) + 2\,C_{0,3}(Q^2) = 0,\\
D_0^{(2)}(Q^2) &:= C_{0,2}(Q^2) + C_{0,4}(Q^2) = 0,\\
D_0^{(3)}(Q^2) &:= C_{0,3}(Q^2) + C_{0,5}(Q^2) = 0.
\end{aligned} \tag{92}$$

Hence it is enough to give $C_{0,4}$ and $C_{0,5}$ here:

$$\begin{aligned}
\left(16\pi^2\right) C_{0,4} = &+\frac{1}{\varepsilon^2}\left\{-\frac{11}{1944}a_s^2 C_A^2 n_g + \frac{109}{15552}a_s^2 n_f C_A T_F n_g - \frac{7}{3888}a_s^2 n_f^2 T_F^2 n_g\right\}\\
&+\frac{1}{\varepsilon}\left\{-\frac{1}{15}n_g - \frac{1}{30}n_f d_R + \frac{1}{54}a_s C_A n_g - \frac{7}{432}a_s n_f T_F n_g - \frac{35}{11664}a_s^2 C_A^2 n_g\right.\\
&\qquad\left.+\frac{1}{192}a_s^2 n_f T_F C_F n_g - \frac{809}{93312}a_s^2 n_f C_A T_F n_g + \frac{77}{23328}a_s^2 n_f^2 T_F^2 n_g\right\}\\
&-\frac{47}{450}n_g - \frac{23}{225}n_f d_R - \frac{1}{15}l_{\mu q} n_g - \frac{1}{30}l_{\mu q} n_f d_R\\
&+a_s\left\{+\frac{187}{1620}C_A n_g - \frac{1987}{12960}n_f T_F n_g + \frac{1}{27}l_{\mu q} C_A n_g - \frac{7}{216}l_{\mu q} n_f T_F n_g\right.\\
&\qquad\left.+\frac{1}{5}\zeta_3 C_A n_g + \frac{1}{10}\zeta_3 n_f T_F n_g\right\}\\
&+a_s^2\left\{+\frac{160831}{1399680}C_A^2 n_g + \frac{61}{480}n_f T_F C_F n_g - \frac{1733639}{2799360}n_f C_A T_F n_g + \frac{140909}{699840}n_f^2 T_F^2 n_g\right.\\
&\qquad+\frac{941}{9720}l_{\mu q} C_A^2 n_g + \frac{1}{64}l_{\mu q} n_f T_F C_F n_g - \frac{15943}{77760}l_{\mu q} n_f C_A T_F n_g + \frac{593}{9720}l_{\mu q} n_f^2 T_F^2 n_g\\
&\qquad-\frac{11}{648}l_{\mu q}^2 C_A^2 n_g + \frac{109}{5184}l_{\mu q}^2 n_f C_A T_F n_g - \frac{7}{1296}l_{\mu q}^2 n_f^2 T_F^2 n_g\\
&\qquad-\frac{5}{12}\zeta_5 C_A^2 n_g - \frac{1}{4}\zeta_5 n_f T_F C_F n_g + \frac{1}{24}\zeta_5 n_f C_A T_F n_g\\
&\qquad+\frac{563}{720}\zeta_3 C_A^2 n_g + \frac{37}{240}\zeta_3 n_f T_F C_F n_g - \frac{29}{720}\zeta_3 n_f C_A T_F n_g - \frac{19}{180}\zeta_3 n_f^2 T_F^2 n_g\\
&\qquad\left.+\frac{11}{60}\zeta_3 l_{\mu q} C_A^2 n_g + \frac{1}{40}\zeta_3 l_{\mu q} n_f C_A T_F n_g - \frac{1}{30}\zeta_3 l_{\mu q} n_f^2 T_F^2 n_g\right\},
\end{aligned} \tag{93}$$

$$
\begin{aligned}
\left(16\pi^2\right) C_{0,5} = &+\frac{1}{\varepsilon^2}\left\{+\frac{11}{1296}a_s^2C_A^2n_g - \frac{109}{10368}a_s^2n_fC_AT_Fn_g + \frac{7}{2592}a_s^2n_f^2T_F^2n_g\right\} \\
&+\frac{1}{\varepsilon}\left\{+\frac{1}{10}n_g + \frac{1}{20}n_fd_R - \frac{1}{36}a_sC_An_g + \frac{7}{288}a_sn_fT_Fn_g - \frac{1}{864}a_s^2C_A^2n_g\right. \\
&\qquad\left.-\frac{1}{128}a_s^2n_fT_FC_Fn_g + \frac{415}{20736}a_s^2n_fC_AT_Fn_g - \frac{35}{5184}a_s^2n_f^2T_F^2n_g\right\} \\
&+\frac{9}{100}n_g + \frac{3}{25}n_fd_R + \frac{1}{10}l_{\mu q}n_g + \frac{1}{20}l_{\mu q}n_fd_R \\
&+a_s\left\{-\frac{1}{540}C_An_g + \frac{1367}{8640}n_fT_Fn_g - \frac{1}{18}l_{\mu q}C_An_g + \frac{7}{144}l_{\mu q}n_fT_Fn_g\right. \\
&\qquad\left.-\frac{3}{10}\zeta_3C_An_g - \frac{3}{20}\zeta_3n_fT_Fn_g\right\} \\
&+a_s^2\left\{+\frac{343429}{933120}C_A^2n_g - \frac{307}{1440}n_fT_FC_Fn_g + \frac{983059}{1866240}n_fC_AT_Fn_g - \frac{109129}{466560}n_f^2T_F^2n_g\right. \\
&\qquad-\frac{67}{12960}l_{\mu q}C_A^2n_g - \frac{3}{128}l_{\mu q}n_fT_FC_Fn_g + \frac{10663}{51840}l_{\mu q}n_fC_AT_Fn_g - \frac{473}{6480}l_{\mu q}n_f^2T_F^2n_g \\
&\qquad+\frac{11}{432}l_{\mu q}^2C_A^2n_g - \frac{109}{3456}l_{\mu q}^2n_fC_AT_Fn_g + \frac{7}{864}l_{\mu q}^2n_f^2T_F^2n_g \\
&\qquad+\frac{5}{8}\zeta_5C_A^2n_g + \frac{3}{8}\zeta_5n_fT_FC_Fn_g - \frac{1}{16}\zeta_5n_fC_AT_Fn_g \\
&\qquad-\frac{563}{480}\zeta_3C_A^2n_g - \frac{37}{160}\zeta_3n_fT_FC_Fn_g + \frac{29}{480}\zeta_3n_fC_AT_Fn_g + \frac{19}{120}\zeta_3n_f^2T_F^2n_g \\
&\qquad\left.-\frac{11}{40}\zeta_3l_{\mu q}C_A^2n_g - \frac{3}{80}\zeta_3l_{\mu q}n_fC_AT_Fn_g + \frac{1}{20}\zeta_3l_{\mu q}n_f^2T_F^2n_g\right\}.
\end{aligned} \tag{94}
$$

A.2 $C_1^{(r)}$, $r = 1\ldots5$

Here we give the five coefficents

$$
\begin{aligned}
C_{1,1} =&a_s\left\{\frac{4}{9}C_A - \frac{7}{18}n_fT_F\right\} \\
&+a_s^2\left\{\frac{3}{8}n_fT_FC_F + \frac{1}{36}n_fC_AT_F - \frac{1}{18}n_f^2T_F^2\right\},
\end{aligned} \tag{95}
$$

$$
\begin{aligned}
C_{1,2} =&a_s\left\{-C_A + \frac{1}{4}n_fT_F\right\} \\
&+a_s^2\left\{-\frac{83}{216}C_A^2 + \frac{25}{48}n_fT_FC_F + \frac{35}{216}n_fC_AT_F + \frac{5}{108}n_f^2T_F^2\right\},
\end{aligned} \tag{96}
$$

$$
\begin{aligned}
C_{1,3} =&a_s\left\{\frac{5}{18}C_A + \frac{5}{72}n_fT_F\right\} \\
&+a_s^2\left\{\frac{83}{432}C_A^2 - \frac{43}{96}n_fT_FC_F - \frac{41}{432}n_fC_AT_F + \frac{1}{216}n_f^2T_F^2\right\},
\end{aligned} \tag{97}
$$

$$
\begin{aligned}
C_{1,4} = &+\frac{1}{3} \\
&\frac{a_s}{\varepsilon}\left\{\frac{11}{36}C_A - \frac{1}{9}n_f T_F\right\} \\
&+\frac{1}{3} + a_s\left\{\frac{161}{216}C_A - \frac{17}{108}n_f T_F\right\} \\
&+\frac{a_s^2}{\varepsilon}\left\{\frac{17}{72}C_A^2 - \frac{1}{12}n_f T_F C_F - \frac{5}{36}n_f C_A T_F\right\} \\
&+a_s^2\left\{\frac{3}{16}C_A^2 - \frac{65}{144}n_f T_F C_F - \frac{5}{108}n_f C_A T_F - \frac{5}{108}n_f^2 T_F^2\right\},
\end{aligned} \tag{98}
$$

$$
\begin{aligned}
C_{1,5} = &-\frac{2}{3} \\
&\frac{a_s}{\varepsilon}\left\{-\frac{11}{18}a_s C_A + \frac{2}{9}a_s n_f T_F\right\} \\
&-\frac{2}{3} + a_s\left\{-\frac{41}{108}C_A - \frac{7}{216}n_f T_F\right\} \\
&+\frac{a_s^2}{\varepsilon}\left\{-\frac{17}{36}C_A^2 + \frac{1}{6}n_f T_F C_F + \frac{5}{18}n_f C_A T_F\right\} \\
&+a_s^2\left\{-\frac{13}{48}C_A^2 + \frac{137}{288}n_f T_F C_F + \frac{61}{432}n_f C_A T_F - \frac{1}{216}n_f^2 T_F^2\right\},
\end{aligned} \tag{99}
$$

which fulfill the relations eq. (90):

$$
\begin{aligned}
D_1^{(1)} =& 0\,, \\
D_1^{(2)} =& \text{local } \neq 0, \\
D_1^{(3)} =& \text{local } \neq 0.
\end{aligned} \tag{100}
$$

This is an important check as for the coeffiicent $C_1^{\mu\nu;\rho\sigma}$ only counterterms of the form $t_4^{\mu\nu;\rho\sigma}[O_1]$ and $t_5^{\mu\nu;\rho\sigma}[O_1]$ are possible. Counterterms proportional to the other tensor structures would not be local. Hence $D_1^{(1)} = 0$ is necessary.

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